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The Bethe-Salpeter equation and the Low Energy Theorems for πN scattering

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Abstract. The Bethe-Salpeter (BS) amplitude for πN scattering is evaluated at the off-mass-shell points corresponding to the Low Energy Theorems (LET) based on PCAC and current algebra. The results suggest a way of maintaining constructing between BS equation and LET.

PACS. 25.80.Dj Pion elastic scattering – 11.30.Rd Chiral symmetries – 24.80.+y Nuclear tests of fundamental interactions and symmetries

1 Introduction

The formulations of πN scattering can be divided into two approaches. On the one hand, we have the Effective Field Theory (EFT) approach where the emphasis is on preserving the symmetries of QCD. This is achieved by the expansion for the amplitude in powers of the pion mass or external momenta divided by a typical QCD cut-off of ≈ 1 GeV. For the πN system, the commonly used EFT is Chiral Perturbation Theory (ChPT) [1] which results in an off-mass-shell amplitude that is consistent with the Low Energy Theorems (LET). These LET are based on current algebra and PCAC. On the other hand, we have the more traditional approach of using the two-body scattering equation (*e.g.*, the Bethe-Salpeter equation) in which the potential is based on s-, t-, and u-channel pole diagrams derived from a chirally invariant Lagrangian. In this case, unitarity is the main uniting feature which allows the examination of πN scattering at higher energies.

2 The Bethe-Salpeter amplitude

With the advent of solutions to the Bethe-Salpeter (BS) equation for the off-mass-shell πN amplitude [2], it is now possible to compare the results of the traditional approach based on two-body scattering, with the LET. Here we will present solutions to the BS equation based on a potential derived from the chirally invariant Lagrangian [2]:

$$\begin{aligned}\mathcal{L}_{\rm int} = &\frac{g_{\pi NN}}{2m_N}\bar\psi_N\gamma_5\gamma^\mu\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}\psi_N \\ &+\frac{f_{\pi N\Delta}}{m_\pi}\bar\psi^\mu_\Delta(g_{\mu\nu}+x_\Delta\gamma_\mu\gamma_\nu)\boldsymbol{T}\psi_N\cdot\partial^\nu\boldsymbol{\pi}+\text{h.c.} \\ &+g_{\rho NN}\bar\psi_N\frac{1}{2}\boldsymbol{\tau}\cdot\left(\gamma^\mu\boldsymbol{\rho}_\mu+\frac{\kappa_\rho}{2m_N}\sigma^{\mu\nu}\partial_\mu\boldsymbol{\rho}_\nu\right)\psi_N \\ &+g_{\rho\pi\pi}\boldsymbol{\rho}_\mu\cdot(\partial^\mu\boldsymbol{\pi}\times\boldsymbol{\pi}) \\ &+g_{\sigma NN}\bar\psi_N\psi_N\sigma \\ &+\frac{g_{\sigma\pi\pi}}{2m_\pi}\sigma\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}\,. \end{aligned} \quad (1)$$

The tree level diagrams that contribute to the potential include the s- and u-channel diagrams with nucleon and Δ poles, and t-channel diagrams with ρ and σ poles. To solve the four-dimensional BS equation, we have introduced form factors for each of the vertices in the Lagrangian. Here we considered two classes of form factors

$$f_{\alpha\beta\gamma}(p^2_\alpha,p^2_\beta,p^2_\gamma)=f_\alpha(p^2_\alpha)\,f_\beta(p^2_\beta)\,f_\gamma(p^2_\gamma)\quad\text{(type I)}\quad(2)$$

and

$$f_{\alpha\beta\gamma}(p^2_\alpha,p^2_\beta,p^2_\gamma)=f_\pi(p^2_\pi)\qquad\text{(type II)}\,,\quad(3)$$

where

$$f_\alpha(p^2_\alpha)=\left(\frac{\Lambda^2_\alpha-m^2_\alpha}{\Lambda^2_\alpha-p^2_\alpha}\right)^{n_\alpha}\,.\quad(4)$$

In the above, m_α and Λ_α are the mass and cut-off associated with the hadron α. The parameters in the Lagrangian are adjusted to fit the s- and p-wave scattering data up to pion laboratory energy of 300 MeV. In fig. 1 we present a

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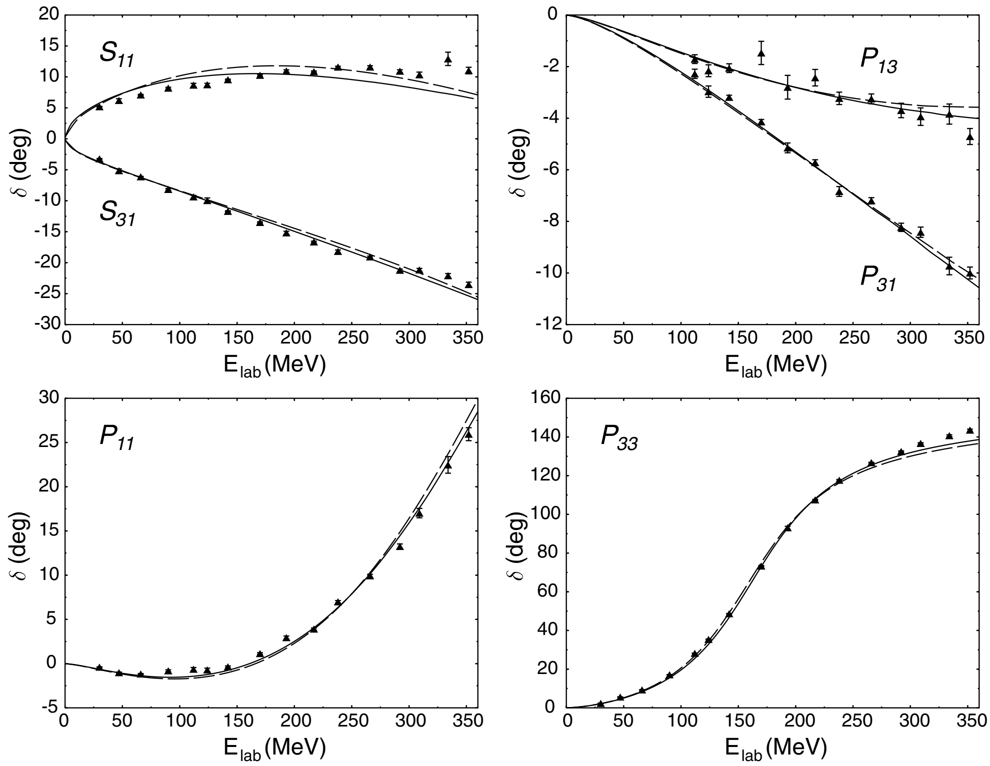

Fig. 1. The S and P wave phase shifts for the form factors type I (solid line) and II (dashed line), the data is SM95 [3].

fit to the data for the form factors type I and II ($n = 4$). Similar fits are achieved for the form factors II ($n = 2$) and II ($n = 10$). Here, the form factors determine the off-mass-shell behaviour of the BS amplitudes. In this way we can vary the off-mass-shell amplitudes when comparing the results of the BS equations with those based on current algebra and PCAC.

3 The Low Energy Theorems

The LET for πN scattering are based on current algebra and PCAC. The latter is implemented by defining the pion field, π^a, in terms of the derivative of the axial vector current, *i.e.* $\partial^\mu A^a_\mu = f_\pi \, m_\pi^2 \; \pi^a$. This allows us to write the πN amplitude in terms of the commutation relation of the currents. In this way we can use current algebra to determine the πN amplitude in the soft pion limit.

The πN amplitude with off-shell pion can be written as

$$T^{ab}_{\pi N} = \bar{u}(p') \left\{ T^{(+)} \delta_{ab} + \tfrac{1}{2} \left[\tau_a, \tau_b \right] T^{(-)} \right\} u(p) \;, \qquad (5)$$

where p (p') is the initial (final) on-shell nucleon momentum, and

$$T^{(\pm)} = D^{(\pm)} + \frac{i}{2m} \sigma^{\mu\nu} q_\mu q'_\nu B^{(\pm)} \;. \qquad (6)$$

Here the $(+)$, $(-)$ refer to isospin even and odd components of the amplitude, and q (q') are the off-mass-shell pion initial (final) momentum. The amplitudes D and B with the pion off mass shell are a function of ν, ν_B, q^2 and q'^2, *i.e.* $D^{(\pm)} = D^{(\pm)}(\nu, \nu_B, q^2, q'^2)$ with $\nu = \frac{1}{4m}(s-u)$ and ν_B being the value of ν at the s-channel nucleon pole, and s and u being the standard Mandelstam variables. Current algebra and PCAC can impose constraints on these off-mass-shell amplitudes. In particular, we can write the isospin even amplitude with the nucleon pole subtracted, *i.e.* $\tilde{D}^+(\nu, \nu_B, q^2, q'^2)$ at three off-shell points. The amplitude at Weinberg (W) [4], Adler (A) [5] and the Cheng-Dashen (CD) [6] points are

$$\tilde{D}^+(0,0,0,0) = -\frac{\sigma_{\pi N}(0)}{f_\pi^2} \;, \qquad (7)$$

$$\tilde{D}^+(0,0,m_\pi^2,0) = 0 = \tilde{D}^+(0,0,0,m_\pi^2) \;, \qquad (8)$$

$$\tilde{D}^+(0,0,m_\pi^2,m_\pi^2) = \frac{\sigma_{\pi N}(0)}{f_\pi^2} + \mathcal{O}(m_\pi^4) + \cdots \;, \qquad (9)$$

respectively. Here we observe that the amplitude at the Weinberg and Cheng-Dashen points are opposite in sign, while that at the Adler point is zero. Since the sigma-term $\sigma_{\pi N}(0)$ is a measure of chiral-symmetry breaking, *i.e.*

$$\sigma_{\pi N}(0) = \frac{1}{2} \sum_{a=1}^{3} \langle N(p) | \, [Q_a^5, [Q_a^5, H]] | N(p) \rangle \;, \qquad (10)$$

in the absence of any mechanism for chiral-symmetry breaking, the πN amplitude is zero at all three points.

Table 1. The coupling constants and masses for the optimum fit to the data for different choices for the form factors. All coupling constants are $g^2/4\pi$.

	I ($n=1$)	II ($n=2$)	II ($n=4$)	II ($n=10$)
$g^{(0)2}_{\pi NN}$	1.80	4.23	4.68	5.98
$f^{(0)2}_{\pi n\Delta}$	0.37	0.17	0.20	0.196
x_Δ	−0.11	−0.13	−0.24	−0.18
$g_{\rho NN}g_{\rho\pi\pi}$	2.88	2.67	2.63	2.80
κ_ρ	2.66	2.18	2.03	2.15
$g_{\sigma\pi\pi}g_{\sigma NN}$	−0.41	0.86	0.39	0.48
$m^{(0)}_N$	1.34	1.18	1.14	1.11
$m^{(0)}_\Delta$	2.305	1.495	1.492	1.435
m_σ	0.65	0.88	0.62	0.64
Λ^{R}_π	1.22	0.874	0.868	0.822
Δ_π	1.3%	2.47%	2.51%	2.79%

Table 2. The BS amplitude at the Adler [5], Weinberg [4] and Cheng-Dashen [6] points in units of m_π^{-1} for different form factors. Also included are the σ-term $\sigma_{\pi N}(0)$ and the isospin-even S-wave scattering length.

Model	A	W	CD	$\sigma_{\pi N}(0)$	a^+
I	0.366	0.355	0.379	23.8	−0.025
II ($n=2$)	0.0949	0.102	0.106	6.64	−0.05
II ($n=4$)	0.0411	0.0462	0.0494	3.10	−0.048
II ($n=10$)	0.0180	0.0218	0.0259	1.62	−0.049

4 Results

To examine the variation in the off-mass-shell BS amplitude when comparing with the results from the LET, we have considered four possible form factors for our potential. In table 1 we have the parameters that give the optimum fit to the data up to pion energy of 300 MeV for the form factor types I ($n=1$), II ($n=2$), and II ($n=4$) and II ($n=10$). Also included in the table are the equivalent cut off mass for a monopole Λ^{R}_π, and the difference in the form factor at the pion pole and at $q^2=0$, *i.e.* $\Delta_\pi = 1 - f^{\mathrm{R}}_\pi(0)$. Here, R refers to the fact that these quantities are calculated for the renormalised form factor. From the table we observe that the dressing of the nucleon and Δ is substantially more for type-I form factors than is the case for type-II form factors. At the same time the type-I form factors give a value for Δ_π that is closer to the commonly accepted value of 3% from the Goldberger-Treiman relation.

In table 2 we present the off-shell amplitude resulting from the solution of the BS equations at the Adler (A), Weinberg (W), and the Cheng-Dashen(CD) points for different choices for our form factor. Also included are the values for the πN σ-term $\sigma_{\pi N}(0)$ and the isospin even scattering length a^+. Here we observe that:

1. The off-mass-shell amplitude is sensitive to the choice of cut off form factor, and in particular, model I gives a larger σ-term than model II.
2. The amplitudes at the three off-mass-shell points are approximately the same. This is in contrast to the fact that the amplitudes at the Weinberg and Cheng-Dashen points are equal and opposite in sign.
3. Finally, the amplitude at the Adler point is not zero.

To understand the difference between the amplitude resulting from the solution of the BS equation and the LET, we first examined the Born amplitude for the πN potentials considered. Here we found that the Born amplitude at all three points is zero, consistent with the requirement of chiral-symmetry conservation. (The σ-term is a measure of chiral-symmetry breaking). This suggests that the higher terms in the multiple-scattering series give all the contribution to the amplitude at the three off-mass-shell points. To examine the source of this chiral-symmetry–breaking contribution, we have examined the contribution of each term in the potential to the amplitude at the Adler, Weinberg and Cheng-Dashen points. This exercise revealed that the source of chiral-symmetry breaking is the higher-order multiple-scattering t-channel ρ exchange. In particular, it is the $\rho\pi\pi$-term in the Lagrangian that gives rise to the symmetry breaking. Since this $\rho\pi\pi$ Lagrangian is of the form $g_{\rho\pi\pi}\,\boldsymbol{\rho}_\mu\cdot(\partial^\mu\boldsymbol{\pi}\times\boldsymbol{\pi})$, then the only time this term in the Lagrangian contributes to chiral-symmetry breaking is when the external pion is represented by $\boldsymbol{\pi}$ rather than $\partial^\mu\boldsymbol{\pi}$. It is possible to restore chiral symmetry to the amplitude by introducing a symmetric form for the coupling of the ρ to the π, *e.g.* $(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu)\cdot(\partial^\mu\boldsymbol{\pi}\times\partial^\nu\boldsymbol{\pi})$ [7], which is equivalent on the mass shell to the form employed in the present calculation. The resultant amplitude would then satisfy chiral symmetry. Chiral-symmetry breaking could then be introduced in a controlled form via a non-derivative $\sigma\pi\pi$ coupling.

5 Conclusion

In the above analysis we have established that the Bethe-Salpeter amplitude for a Lagrangian that satisfies chiral symmetry is inconsistent with the Low Energy Theorems of current algebra. The source of the disagreement is the mode of chiral-symmetry breaking via the higher-order multiple scattering of t-channel ρ exchange. This could be overcome with the introduction of a symmetric $\rho\pi\pi$ Lagrangian, and a mode of chiral-symmetry breaking that is consistent with the LET.

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